Name

Summer Assignment for Calculus II – MATH 116
(for those who just finished Calculus I)

SHOW YOUR WORK when doing each problem. Here are the Calculus 1 Notes for reference.
Only do Exercises #0a-0d if you feel like you struggle with trig identities or factoring.

Exercise 0a:

Simplify the expression $\frac{\tan(x)}{\csc(x)} + \frac{\sin(x)}{\tan(x)}$.

a. $\sec(x)$	c. $\sec^2(x)$
b. $\csc^2(x)$	d. $\cos(x)$

Exercise 0b:

Simplify the expression $\frac{\cos(x)\sec(x)}{\cot(x)}$.

a. $\sin(x)$	c. $\cot(x)$
b. $\tan(x)$	d. $\cos(x)$

Exercise 0c:

Simplify the expression cos(x) + sin(x) tan(x).

a. $\cos(x)$	c. $\cot(x)$
b. $\sec(x)$	d. $\csc(x)$

Exercise 0d:

Solve $e^{2x} - 3e^{x} + 2 = 0$ for x.

a. $x = 0, \ln(2)$	c. $x = 0, \ln(5)$
b. $x = 0, \ln(3)$	d. $x = 1, 2$

For Exercises 1-4, use the unit circle or special right triangles to solve these problems. NO CALCULATOR. SHOW YOUR WORK using the unit circle or special right triangles.

Exercise 1:

If $\sin(x) = \frac{\sqrt{3}}{2}$ for x in Quadrant I, find $\tan(x) + \sec(x)$.

a. $\sqrt{3} + 2$	c. $\sqrt{3} + 1$
b. $\sqrt{2} + 3$	d. $\sqrt{3} - 3$

Exercise 2:

Given cot(x) is undefined and cos(x) > 0, find csc(x).

a.) 1	c.) Undefined
b.) 0	d.) -1

Exercise 3:

Find all solutions in the interval $[0, 2\pi)$ of $2\sin(x) - \sqrt{2} = 0$.

a. $x = \frac{\pi}{4}, -\frac{\pi}{4}$	c. $x = \frac{\pi}{3}, \frac{2\pi}{3}$
b. $x = \frac{\pi}{2}, \frac{3\pi}{2}$	d. $x = \frac{\pi}{4}, \frac{3\pi}{4}$

Exercise 4:

Find the exact value of $\tan^{-1}\frac{\sqrt{3}}{3}$.

Exercise 5:

Find numbers a and b, or k, so that f is continuous at every point.

$$f(x) = \begin{cases} -7, & x < -4 \\ ax + b, & -4 \le x \le 3 \\ 21, & x > 3 \end{cases}$$

For Exercises 6-14: find the limits. You may use L'Hopital's Rule when necessary.
See Sections 2.3, 2.4, 2.5, and 4.7 for notes:

Exercise 6: $\lim\limits_{x \to 0} \dfrac{x^3 - 6x + 8}{x - 2}$	Exercise 7: $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x}$
Exercise 8: $\lim\limits_{x \to 1^-} \dfrac{x^2 - 4x + 3}{x^3 - x}$	Exercise 9: $\lim\limits_{x \to -2^+} \left(\dfrac{x^2 - 7x + 10}{x^3 - 4x} \right)$
Exercise 10: $\lim\limits_{x \to \infty} \left(\dfrac{2x^3 - 5x^2 + 3x}{-x^3 - 2x + 7} \right)$	Exercise 11: Find all the vertical asymptotes, and then **use limits** to find all the horizontal asymptotes of $f(x) = \dfrac{x - 1}{x^3 + 5x^2 - 84x}$.
Exercise 12: $\lim\limits_{x \to (-\pi/2)^-} \sec x$	Exercise 13: $\lim\limits_{x \to 0} \dfrac{e^x - \sin x - 1}{x^4 + 8x^3 + 12x^2}$
Exercise 14: $\lim\limits_{y \to 2} \dfrac{y^2 + y - 6}{\sqrt{8 - y^2} - y}$	

For Exercises 15-16: Divide numerator and denominator by the highest power of x in the denominator to find the limit.

Exercise 15: $\lim\limits_{x \to \infty} \sqrt{\dfrac{25x^2}{6 + 16x^2}}$	Exercise 16: $\lim\limits_{x \to \infty} \dfrac{5x + 6}{\sqrt{6x^2 + 1}}$

For the next exercise, see Section 2.5 Limits at Infinity + 2.3 Squeeze Theorem for notes:

Exercise 17:

The inequality $1-\frac{x^2}{2} < \frac{\sin x}{x} < 1$ holds when x is measured in radians and $|x| < 1$.

Find $\lim_{x \to 0} \frac{\sin x}{x}$ if it exists.

For the next exercise, see Section 2.6 Continuity for notes:

Exercise 18:

Use the Intermediate Value Theorem to prove that $x(x - 2)^2 = 2$ has a solution between 1 and 3.

Exercise 19:

Use the limit definition of the derivative $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ to find f '(a) given the following:

$$f(x) = \frac{1}{3x - 1}; \; a = 2$$

Exercise 20:

Use the limit definition of the derivative $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ to find f '(a) given the following:

$$f(x) = \frac{x}{x + 1};$$

a = -2

Exercise 21:

Use the limit definition of the derivative $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ to find f '(a) given the following:

$$f(x) = \frac{1}{\sqrt{x}}; \; a = \frac{1}{4}$$

Exercise 22:

The function s = f(t) gives the position of a body moving on a coordinate line, with s in meters and t in seconds.

$s = 9t - t^2, \; 0 \le t \le 9$

Find the body's displacement and average velocity for the given time interval.

Exercise 23:

At time t, the position of a body moving along the s-axis is $s = t^3 - 21t^2 + 120t$ m. Find the body's acceleration each time the velocity is zero.

For Exercises 24-27, find dy/dt. You may need to use the Chain Rule, Product Rule, Quotient Rules, and other derivative rules.

Exercise 24:	Exercise 25:
$y = \cos^4(\pi t - 17)$	$y = 5t(2t + 4)^4$

Exercise 26:	Exercise 27:
$y = \dfrac{te^t}{t + 1}$	$y = 2\sqrt{t}$

Exercise 28:	Exercise 29:
Find the tangent line to the graph of $f(x) = -8e^{5x}$ at the point (0, −8).	**Use implicit differentiation to to find dy/dx and d^2y/dx^2.** $xy - x + y = 6$

Exercise 30:	31.) Use logarithmic differentiation to find the derivative of y with respect to the independent variable.
At the given point, find the line that is normal to the curve at the given point. Use implicit differentiation. $x^6y^6 = 64$, normal at (2, 1)	$y = (x + 10)^x$

For #32-33, solve the related rates problems.

32.) A wheel with radius 3 m rolls at 15 rad/s. How fast is a point on the rim of the wheel rising when the point is π/3 radians above the horizontal (and rising)? (Round your answer to one decimal place.)

33.) A piece of land is shaped like a right triangle. Two people start at the right angle of the triangle at the same time, and walk at the same speed along different legs of the triangle. If the area formed by the positions of the two people and their starting point (the right angle) is changing at 2 m^2/s, then how fast are the people moving when they are 3 m from the right angle? (Round your answer to two decimal places.)

For #34, solve the related rates problem.

34.) Boyle's law states that if the temperature of a gas remains constant, then PV = c, where P = pressure, V = volume, and c is a constant. Given a quantity of gas at constant temperature, if V is decreasing at a rate of 15 $in.^3$/sec, at what rate is P increasing when P = 40 $lb/in.^2$ and V = 80 $in.^3$? (Do not round your answer.)

35.) First, find the x-intercepts of the graph. Then use the First Derivative Test and the Second Derivative Test to find local extrema (local maximums and minimums) and points of inflection. SHOW YOUR WORK, and then sketch the graph with these features for the following equation:

$$f(x) = x^4 - 6x^2$$

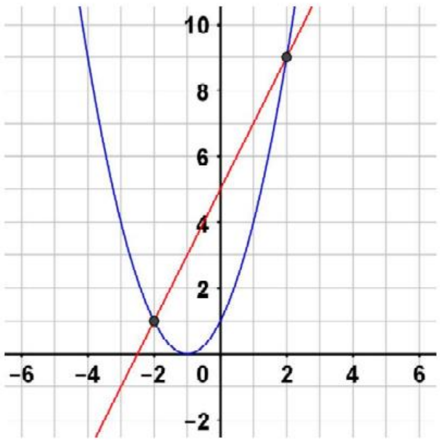
For Exercises 36-40, use U-Substitution to solve the following definite integrals.

Exercise 36: $\int_0^1 2x(4 - x^2)\,dx$	Exercise 37: $\int_0^{\pi/2} \sin^2\theta\cos\theta\,d\theta$
Exercise 38: $\int_{-1}^{2} x^2 e^{x^3+1}\,dx$	Exercise 39: $\int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}\,d\theta$
Exercise 40: $\int_0^3 \frac{v^2 + 1}{\sqrt{v^3 + 3v + 4}}\,dv$	

For Exercises #41-42, set up the definite integral that gives the area of the region between the curves. See Section 6.2: Regions Between Curves, for notes:

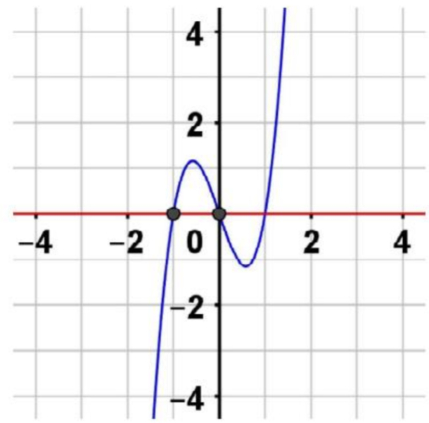
Exercise 41:

$y_1 = x^2 + 2x + 1, \quad y_2 = 2x + 5$



Exercise 42:

$y_1 = 3\left(x^3 - x\right), \quad y_2 = 0$



For Exercises 43-44, see Sections 6.3-6.4: Volume by Slicing (Disks + Washers) + Volume by Shell - for notes:

Exercise 43:

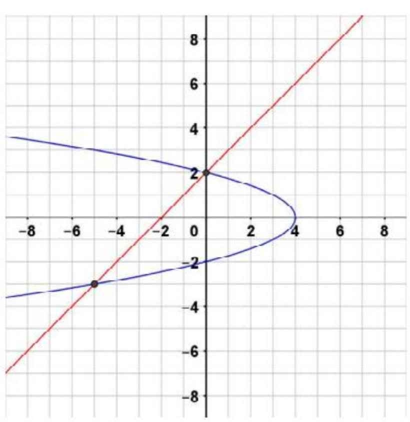
Find the area of the region between the two functions by integrating

(a) with respect to x *and*

(b) with respect to y.

(c) Compare your results. Which method is simpler?

$x = 4 - y^2, \quad x = y - 2$



Exercise 44: Find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

$y = 2x^2, \quad y = 0, \quad x = 2$

a.) the y-axis
b.) the x-axis
c.) the line y = 8
d.) the line x = 2

Exercises 45-46: Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

Exercise 45:

$y = -x^3 + 3, \quad y = x, \quad x = -1, \quad x = 1$

Exercise 46:

$f(x) = -x^2 + 4x + 1, \quad g(x) = x + 1$