Answers: Summer Assignment for Calculus II – MATH 116

Exercise 0a: A

Exercise 0b: B

Exercise 0c: B

Exercise 0d: A

Exercise 1: A

Exercise 2: C

Exercise 3: D

Exercise 4: π/6

Exercise 5:
a = 4, b = 9

Exercise 6: 4

Exercise 7: -1/2

Exercise 8: -1

Exercise 9: ∞

Exercise 10: -2

Exercise 11:
horizontal asymptote: $y = 0$;
vertical asymptotes: $x = 0, -12, 7$

Exercise 12: ∞

Exercise 13: 1/24

Exercise 14: -5/2

Exercise 15:
$\frac{5}{4}$

Exercise 16:
$\frac{5}{\sqrt{6}}$

Exercise 17: 1

Exercise 18:
Let $f(x) = x(x - 2)^2$ and let $y_0 = 2$. $f(1) = 1$ and $f(3) = 3$. Since f is continuous on [1, 3] and since $y_0 = 2$ is between f(1) and f(3), by the Intermediate Value Theorem, there exists a c in the interval (1, 3) with the property that f(c) = 2. Such a c is a solution to the equation $x(x - 2)^2 = 2$.

Exercise 19:
$$\lim_{h\to 0} -\frac{3}{(5+3h)5} = -\frac{3}{25}.$$

Exercise 20:
$$\lim_{h\to 0} -\frac{1}{-2+h+1} = 1.$$

Exercise 21:
Hint: Combine the complex fractions, then rationalize the numerator.
$$\lim_{h\to 0} -\frac{4}{\sqrt{\frac{1}{4}+h}\left(1+2\sqrt{\frac{1}{4}+h}\right)} = -4.$$

Exercise 22:
0 m, 0 m/sec

Exercise 23:
$a(10) = 18$ m/sec^2, $a(4) = -18$ m/sec^2

Exercise 24:
$-4\pi \cos^3(\pi t - 17)\sin(\pi t - 17)$

Exercise 25:
$5(2t + 4)^3(10t + 4)$

Exercise 26:
$$\frac{e^t(t^2+t+1)}{(t+1)^2}$$

Exercise 27:
$$\frac{\ln 2}{2\sqrt{t}} 2^{\sqrt{t}}$$

Answers: Summer Assignment for Calculus II – MATH 116 (continued):

Exercise 28:

$$y = -40x - 8$$

Exercise 29:

$$\frac{dy}{dx} = \frac{1 - y}{1 + x};\ \frac{d^2y}{dx^2} = \frac{2y - 2}{(x + 1)^2}$$

Exercise 30:

$$y = 2x - 3$$

Exercise 31:

$$(x + 10)^x\left(\ln(x + 10) + \frac{x}{x + 10}\right)$$

Exercise 32:

22.5 m/s

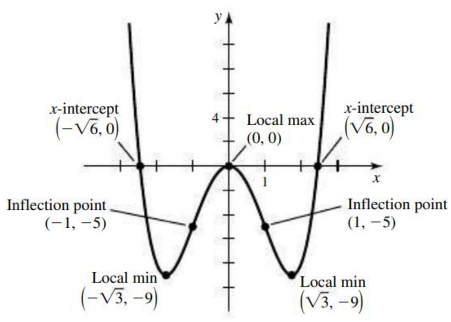
Exercise 33:

0.67 m/s

Exercise 34:

$\frac{15}{2}$ lb/in.^{2} per sec

Exercise 35:



Exercise 36: 3.5

Exercise 37: 1/3

Exercise 38:

$$\frac{e^9 - 1}{3}$$

Exercise 39: 1/2

Exercise 40:

$$\frac{4\sqrt{10} - 4}{3}$$

Exercise 41:

$$A = \int_{-2}^{2} \left[(2x + 5) - \left(x^2 + 2x + 1\right) \right] dx$$

$$= \int_{-2}^{2} \left(-x^2 + 4\right) dx$$

Exercise 42:

$$A = 2\int_{-1}^{0} 3\left(x^3 - x\right) dx$$

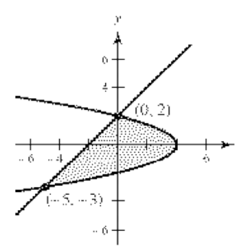
$$= 6\int_{-1}^{0} \left(x^3 - x\right) dx$$

Exercise 43:

a)

$$x = 4 - y^2$$
$$x = y - 2$$
$$4 - y^2 = y - 2$$
$$y^2 + y - 6 = 0$$
$$(y + 3)(y - 2) = 0$$

Intersection points $(0, 2)$ and $(-5, -3)$



$$A = \int_{-5}^{0} \left[(x + 2) + \sqrt{4 - x} \right] dx + \int_{0}^{4} 2\sqrt{4 - x}\ dx$$

$$= \frac{61}{6} + \frac{32}{3} = \frac{125}{6}$$

b) $A = \int_{-3}^{2} \left[\left(4 - y^2\right) - \left(y - 2\right) \right] dy = \frac{125}{6}$

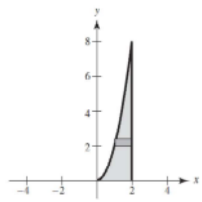
c) The second method is simpler. Explanations will vary.

Answers: Summer Assignment for Calculus II – MATH 116 (continued):

Exercise 44:

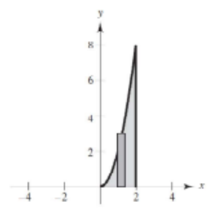
a) $R(y) = 2,\ r(y) = \sqrt{\frac{y}{2}}$

$$V = \pi\int_0^8 \left(4 - \frac{y}{2}\right)dy = \pi\left[4y - \frac{y^2}{4}\right]_0^8 = 16\pi$$



b) $R(x) = 2x^2,\ r(x) = 0$

$$V = \pi\int_0^2 4x^4\,dx = \pi\left[\frac{5x^5}{5}\right]_0^2 = \frac{128\pi}{5}$$



c) $R(x) = 8,\ r(x) = 8 - 2x^2$

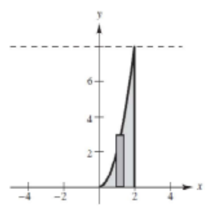
$$V = \pi\int_0^2 \left[64 - \left(64 - 32x^2 + 4x^4\right)\right]dx$$

$$= \pi\int_0^2 \left(32x^2 - 4x^4\right)dx$$

$$= 4\pi\int_0^2 \left(8x^2 - x^4\right)dx$$

$$= 4\pi\left[\frac{8}{3}x^3 - \frac{1}{5}x^5\right]_0^2$$

$$= \frac{896\,\pi}{15}$$

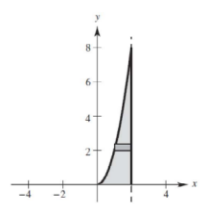


d) $R(y) = 2 - \sqrt{\frac{y}{2}},\ r(y) = 0$

$$V = \pi\int_0^8 \left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$$

$$= \pi\int_0^8 \left(4 - 4\sqrt{\frac{y}{2}} + \frac{y}{2}\right)^2 dy$$

$$= \pi\left[4y - \frac{4\sqrt{2}}{3}y^{\frac{3}{2}} + \frac{y^2}{4}\right]_0^8 = \frac{16\,\pi}{3}$$

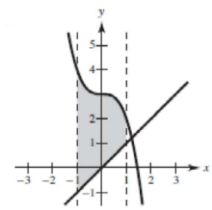


Exercise 45:

$$A = \int_{-1}^1 \left[\left(-x^3 + 3\right) - x\right]dx$$

$$= \left[\frac{-x^4}{4} + 3x - \frac{x^2}{2}\right]_{-1}^1$$

$$= \left(-\frac{1}{4} + 3 - \frac{1}{2}\right) - \left(-\frac{1}{4} - 3 - \frac{1}{2}\right) = 6$$



Exercise 46:

The points of intersection are given by:

$$-x^2 + 4x + 1 = x + 1$$

$$-x^2 + 3x = 0$$

$$x^2 = 3x \quad \text{when } x = 0,\ 3$$

$$A = \int_0^3 \left[\left(-x^2 + 4x + 1\right) - (x + 1)\right]dx$$

$$= \int_0^3 \left(-x^2 + 3x\right)dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3 = -9 + \frac{27}{2} = \frac{9}{2}$$